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A. Verganelakis: POSSIBLE DETERMINATION OF THE CHARACTER OF THE HIGHER RESONANCES IN PION-PHOTO_{PRODUCTION} AND PROTON COM_{PTON} EFFECT BY USING POLARIZED γ RAYS.

Recently many phenomenological analyses of experimental data on pion photoproduction and pion-nucleon scattering indicate that the structures, conventionally called second and third resonances, are more complicated than simple resonances in the $D_{3/2}$ and $F_{5/2}$, $T=1/2$ states respectively^(1,5).

However the experimental information is insufficient for a complete and unambiguous analysis in terms of angular momentum states, and it turns out that different combinations of multipoles may fit equally well the existing experimental data⁽⁴⁾.

This uncertainty has led to some confusion in our understanding of these structures, and more restrictive experimental information is necessary for its clarification.

The purpose of the present note is to point out that, in the case of photoproduction and proton compton effect, ex-

periments with linearly polarized γ 's^(*) would be very helpful for the understanding of the above phenomena⁽⁷⁾.

A. The c.m differential cross section for pion photoproduction by polarized photons has been written down in terms of angular momentum states⁽⁸⁾.

For $\theta_{c.m} = 90^\circ$ (θ is the center of mass angle between the incoming photon direction and the outgoing pion) and including partial waves up to D wave it is given by:

$$\begin{aligned}
 \frac{d\sigma}{d\Omega}_{\text{polarized}} = k R_e \{ & [|E_{01}|^2 + |M_{11}|^2 + \frac{5}{2} |M_{13}|^2 + \frac{9}{2} |E_{13}|^2 + \frac{9}{2} |M_{23}|^2 + \\
 & + \frac{5}{2} |E_{25}|^2 + \frac{9}{2} |M_{25}|^2 + \frac{45}{4} |E_{25}|^2 + 3(E_{01}^* M_{23}) - (E_{01}^* E_{23}) - 3(E_{01}^* M_{25}) - \\
 & - 6(E_{01}^* E_{25}) + (M_{11}^* M_{13}) + 3(M_{11}^* E_{13}) - 3(M_{13}^* E_{13}) + 3(M_{23}^* E_{23}) - 9(M_{23}^* M_{25}) - \\
 & - \frac{9}{2}(M_{23}^* E_{25}) - 3(E_{23}^* M_{25}) + \frac{15}{2}(E_{23}^* E_{25}) + \frac{9}{2}(M_{25}^* E_{25})] + [-\frac{3}{2} |M_{13}|^2 + \frac{9}{2} |E_{13}|^2 - \\
 & - \frac{9}{2} |M_{23}|^2 + \frac{3}{2} |E_{23}|^2 - \frac{9}{2} |M_{25}|^2 + 9 |E_{25}|^2 - 3(E_{01}^* M_{23}) - 3(E_{01}^* E_{23}) + 3(E_{01}^* M_{25}) - \\
 & - 3(E_{01}^* E_{25}) - 3(M_{11}^* M_{13}) + 3(M_{11}^* E_{13}) - 3(M_{13}^* E_{13}) - 3(M_{23}^* E_{23}) + 9(M_{23}^* M_{25}) + \\
 & + \frac{9}{2}(M_{23}^* E_{25}) + 3(E_{23}^* M_{25}) + \frac{21}{2}(E_{23}^* E_{25}) - \frac{9}{2}(M_{25}^* E_{25})] \cos 2\phi \}
 \end{aligned}
 \tag{1}$$

(*) - At present polarized γ 's are available only at low energies, but soon they will also be available for higher energies⁽⁶⁾.

where K is a kinematical factor and ϕ is the angle between the photon polarization vector and the production plane.

The notation is $E_{1\pi 2J}$ for electric and $M_{1\pi 2J}$ for magnetic multipoles. We will consider the two cases of photon polarization - polarization perpendicular ($\phi = 90^\circ$) or parallel ($\phi = 0^\circ$) to the production plane - and we study the corresponding differential cross sections $d\sigma_{\perp}/d\Omega$ and $d\sigma_{\parallel}/d\Omega$. Let us for the moment make the unrealistic assumption that for a certain energy only one of the multipoles contributes.

Then for each multipole we have the results given in table I.

TABLE I

Multipoles	$j_{\gamma}^{(*)}$	$d\sigma_{\parallel}/d\Omega$	$d\sigma_{\perp}/d\Omega$	$d\sigma_{\parallel}/d\sigma_{\perp}$
E01	1	Yes	Yes	1
E13	2	Yes	No	∞
E23	1	Yes	Yes	4
E25	3	Yes	Yes	9
M11	1	Yes	Yes	1
M13	1	Yes	Yes	1/4
M23	2	No	Yes	0
M25	2	No	Yes	0

(*) - j_{γ} total angular momentum of the photon.

From this table we see the following: a) in the ratio $d\sigma_{\parallel} / d\sigma_{\perp}$ the behavior of magnetic multipoles is completely different from the corresponding electric ones (apart from E01 and M11). b) At the region of energy where the state M13 or the state E23 is the only one present the ratio $d\sigma_{\parallel} / d\sigma_{\perp}$ is a constant.

Also $d\sigma_{\parallel} / d\sigma_{\perp} \text{ (M13)} = d\sigma_{\perp} / d\sigma_{\parallel} \text{ (E23)}$.

Based on this table we indicate in Fig. 1, the dashed lines (1) and (2) on which the experimental points for the ratio $d\sigma_{\parallel} / d\sigma_{\perp}$ should lie, for pion photoproduction if it proceeds only through the state E23 or only through the state M13 respectively. Now in what follows we will restrict ourselves to π^0 - photoproduction for simplicity.

It has been pointed out that the resonance at 330 MeV is due to a magnetic dipole M13. A recent experiment at Frascati⁽⁹⁾ has measured the ratio $d\sigma_{\parallel} / d\sigma_{\perp}$ at $\theta \text{ c.m.} = 90^\circ$ and $E_{\gamma L} = 330 \text{ MeV}$ for π^0 - photoproduction and the results fit the table for M13 (see Fig. 1), which indicates clearly once more that at this energy all other partial waves are insignificant by comparison.

At the second resonance ($E_{\gamma L} \simeq 750 \text{ MeV}$) experimental data for the ratio $d\sigma_{\parallel} / d\sigma_{\perp}$ does not exist. If this resonance is due to a pure E23 resonant state⁽¹¹⁾, (*) one would expect the experimental points for $d\sigma_{\parallel} / d\sigma_{\perp}$ to reach the cross-hatched

(*) - This is not in disagreement with the observed polarization of the recoil proton in π^0 -photoproduction⁽¹⁰⁾.

area α . Now if this expected value of $d\sigma_{\parallel}/d\sigma_{\perp}$ is not confirmed by future experiments but is found to be lower than 4, then it will suggest the existence of a background of states of magnetic type and possibly a E01 wave contribution. In that case, in order to clarify the situation, the angular distributions of $d\sigma_{\parallel}/d\Omega$, $d\sigma_{\perp}/d\Omega$, $d\sigma_{\parallel}/d\sigma_{\perp}$ and $(d\sigma_{\parallel} + d\sigma_{\perp})/2 = d\sigma/d\Omega$ at the energy of the second resonance will be very helpful.

To understand this, one has to take into account the fact that the states M23 and M25 do not contribute to $d\sigma_{\parallel}/d\Omega$ and the state E13 does not contribute to $d\sigma_{\perp}/d\Omega$ (see table I). Furthermore it seems reasonable to assume that F waves coming from the third resonance do not affect significantly the structure of the second⁽²⁾, and that the E25 (electric octupole) gives insignificant contribution to the second resonance.

Bearing in mind these points, one may be able to discriminate between the various contributions by comparing the above mentioned angular distributions. For example if a $D_{5/2}$ wave plays a certain role, as recent experimental data of Beneventano et al. indicates⁽¹⁾, then one can recognize it by the difference in behaviour of $d\sigma_{\parallel}/d\Omega$ and $d\sigma_{\perp}/d\Omega$. In the case that the multipole M25 predominates above all others then $d\sigma_{\parallel}/d\Omega \rightarrow 0$ and the $d\sigma_{\perp}/d\Omega$ should be approximately twice the $d\sigma/d\Omega$ at ~ 750 MeV.

Similar considerations apply also to the third resonance.

B. For further confirmation of the nature of the resonances we consider now the case of the proton Compton effect. If we write the center-of-momentum amplitude for the scattering of ph_0

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tons by nucleons in the form⁽¹⁴⁾:

$$\begin{aligned}
 A_{\gamma \rightarrow \gamma} = & R_1(\vec{e}_1, \vec{e}_2) + R_2(\vec{k}_1 \times \vec{e}_1, \vec{k}_2 \times \vec{e}_2) + iR_3(\vec{\sigma} \cdot \vec{e}_2 \times \vec{e}_1) + iR_4(\vec{\sigma} \cdot [\vec{k}_2 \times \vec{e}_2] \times [\vec{k}_1 \times \vec{e}_1]) + \\
 (2) \quad & + iR_5[(\vec{\sigma} \cdot \vec{e}_1)(\vec{k}_2 \cdot \vec{e}_2 \times \vec{e}_1) - (\vec{\sigma} \cdot \vec{k}_2)(\vec{k}_1 \cdot \vec{e}_1 \times \vec{e}_2)] + \\
 & + iR_6[(\vec{\sigma} \cdot \vec{k}_2)(\vec{k}_2 \cdot \vec{e}_2 \times \vec{e}_1) - (\vec{\sigma} \cdot \vec{k}_1)(\vec{k}_1 \cdot \vec{e}_1 \times \vec{e}_2)]
 \end{aligned}$$

and calculate the differential cross section of this process using polarized photons at θ c.m. = 90° we have⁽¹⁵⁾:

$$\begin{aligned}
 \frac{d\sigma}{d\Omega}_{\text{polarized}} = & \frac{1}{2} \{ |R_1|^2 + |R_2|^2 + 3|R_3|^2 + 3|R_4|^2 + 2|R_5|^2 + 2|R_6|^2 + 4\text{Re}(R_3^* R_6 + R_4^* R_5) \} - \\
 (3) \quad & - [|R_1|^2 + |R_4|^2 - |R_2|^2 - |R_3|^2] \cos 2\phi
 \end{aligned}$$

where ϕ now is the angle between the photon polarization vector and the scattering plane.

The expression for R_i at θ c.m. = 90° in terms of multiple amplitudes including those up to $J = 3/2$ can be written⁽¹⁴⁾:

$$R_1 = \mathcal{E}_1 + 2\mathcal{E}_3 - m_2$$

$$R_3 = \mathcal{E}_1 - \mathcal{E}_3 + \frac{1}{2} m_2 + \sqrt{6} C'(m_3, m_2)$$

$$R_5 = -\mathcal{E}_2 - \sqrt{6} C'(m_3, \mathcal{E}_2)$$

The corresponding expressions for R_2, R_4, R_6 may be obtained from those for R_1, R_3, R_5 respectively by the substitution

$\mathcal{E}_i \leftrightarrow m_i$. By \mathcal{E}_1 and \mathcal{E}_3 we denote the amplitudes for electric dipole transitions, $J = 1/2$ and $J = 3/2$, respectively, and by \mathcal{E}_2

the amplitude for the electric quadrupole transition $J = 3/2$.

Likeuise m_1, m_3 and m_2 indicate the magnetic dipole and quadrupole transitions. The $C'(m_k, \mathcal{E}_i)$ are the amplitudes for the transitions from the states m_k to \mathcal{E}_i .

If we again make the assumption that for a certain energy only one of the multipoles contribute then the eqs. 3 and 4 imply the results given in table II.

TABLE II (*)

Multipoles	$d\sigma_{\parallel}/d\Omega$	$d\sigma_{\perp}/d\Omega$	$d\sigma_{\parallel}/d\sigma_{\perp}$
\mathcal{E}_1	Yes	Yes	1
\mathcal{E}_2	Yes	Yes	5/2
\mathcal{E}_3	Yes	Yes	2/5
m_1	Yes	Yes	1
m_2	Yes	Yes	2/5
m_3	Yes	Yes	5/2

These results are independent of the model that one may use to calculate these multipoles and are only a consequence of general properties of the amplitude. However, due to the connection of the proton Compton effect with the pion - photo-production through the unitarity condition one having appropriate experimental data from the first may extract supplementary information about the character of the latter, and vice versa.

In Fig. 2 we indicate the dashed lines (1) and (2) on which the experimental points for the ratio $d\sigma_{\parallel}/d\sigma_{\perp}$ should lie,

(*) - Due to time reversal invariance the above results for $d\sigma_{\parallel}/d\sigma_{\perp}$ are valid also if one uses an unpolarized beam but measures the final photon polarization.

for the proton Compton effect if it proceeds through only the state m_3 or through only the state ξ_3 respectively.

It is to be noticed that if in the proton Compton effect essential divergences are observed different from the behaviour which the photoproduction implies, then it will suggest that effects coming from the crossed channels are important^(*).

Finally we want to stress that the remarks made here indicate only some of the possibilities, to which the above analysis may be applied, in the study of the structure of resonances occurring in the photoproduction and proton Compton effect.

A discussion with professors B. De Tollis, G. Salvini and Dr. D. Zwanziger is gratefully acknowledged.

(*) - Quite recent experimental data in proton Compton effect with unpolarized γ 's at energies that correspond to the second resonance in photoproduction, suggest that the scattering at $\theta = 90^\circ$ proceeds largely through a (1,3) isobar⁽¹⁶⁾. But one is unable from the data to extract information regarding the details of this structure.

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FIGURE CAPTIONS.

Fig. 1. The ratio $d\sigma_{\parallel}/d\sigma_{\perp}$ vs. $E_{\gamma L}$ for pion-photoproduction for $\theta.c.m = 90^{\circ}$. Exp. points should lie on the dashed line (1) if the photoproduction proceeds through only the state E23, or on the dashed line (2) if it proceeds through only the state M13. The exp. points at 750 MeV should lie on the cross-hatched area α if the second resonance is due to a pure E23 state. The reported exp. data at 285, 320 and 435 MeV are taken from ref. 13, 9 and 12, respectively.

Fig. 2. The ratio $d\sigma_{\parallel}/d\sigma_{\perp}$ vs. $E_{\gamma L}$ for proton Compton effect for $\theta.c.m = 90^{\circ}$. Exp. points should lie on the dashed line (1) if the proton Compton effect proceeds through a pure m_3 state, or on the dashed line (2) if it proceeds through a pure $\underline{\epsilon}_3$ state.

TABLE CAPTIONS.

Table 1. Values of $d\sigma_{\parallel}/d\sigma_{\perp}$ for pion-photoproduction assuming that only one multipole term indicated in the left hand column contributes. Non-zero and zero contributions to $d\sigma_{\parallel}/d\Omega$ and $d\sigma_{\perp}/d\Omega$ are indicated by "Yes" and "No" respectively.

Table 2. Values of $d\sigma_{\parallel}/d\sigma_{\perp}$ for proton Compton effect assuming that only one multipole term indicated in the left hand column contributes. By "Yes" is indicated non zero contributions to $d\sigma_{\parallel}/d\Omega$ and $d\sigma_{\perp}/d\Omega$.

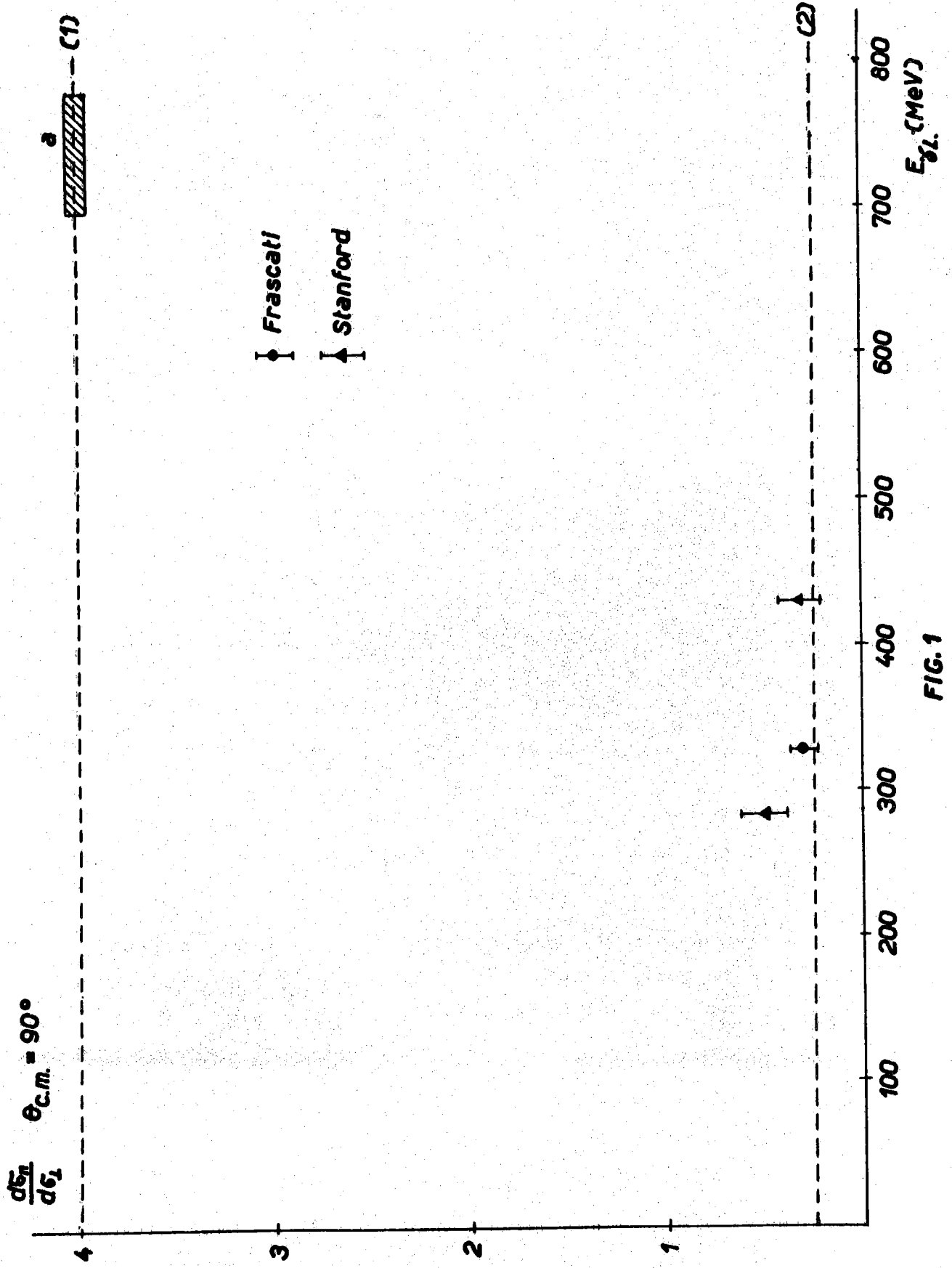


FIG. 1

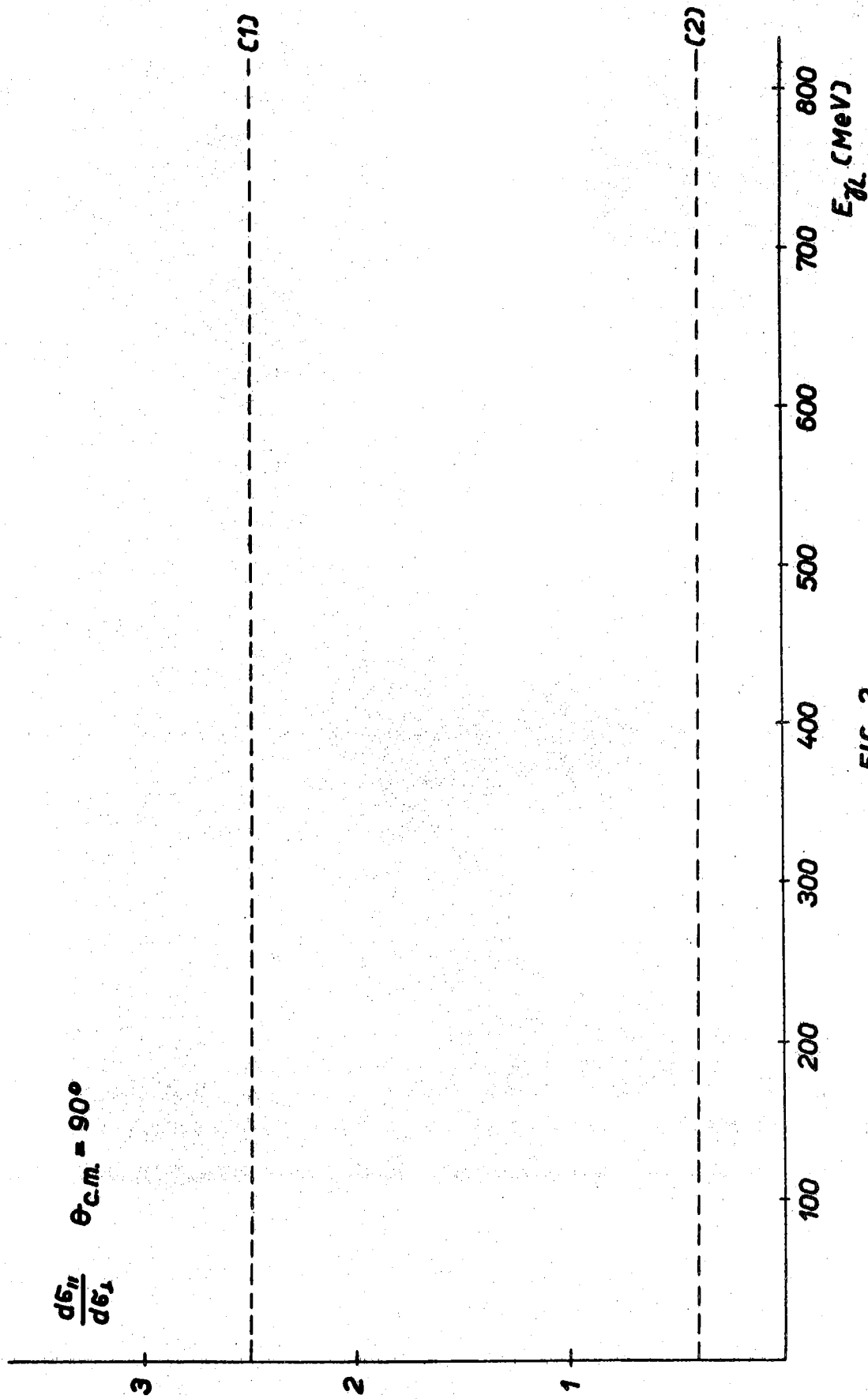


FIG. 2